

Questions	Answer from MMaDA
<p>Question:</p> <p>How many plums will balance one pear, given that 3 apples and one pear weigh as much as 10 plums, and one apple and 6 plums balance one pear? Assume that fruits of the same kind have the same weight.</p>	<p><think> Okay, let's see. I need to figure out how many plums balance one pear. The problem gives me two equations involving apples, pears, and plums. All fruits of the same type weigh the same. Hmm, let's break it down step by step. First, let me restate the information given. The problem says: 1. 3 apples and one pear weigh the same as 10 plums. 2. 1 apple and 6 plums balance one pear. I need to find out how many plums equal one pear. Let me assign variables to each fruit to make it easier. Let's let A represent the weight of an apple, P represent the weight of a pear, and M represent the weight of a plum. Since all fruits of the same kind are identical in weight, these variables should work. So translating the problem into equations: 1) $3A + P = 10M$ 2) $A + 6M = P$ The goal is to find how many plums equal one pear, so I need to express P in terms of M. So essentially, solve for P and find out how many plums one pear equals. Looking at the two equations, maybe I can substitution or elimination. Let's see. Equation 2 gives me P in terms of A and M. If I can substitute P into equation 1, maybe I can solve for A in terms of M, and then substitute that back to find P in terms of M. Starting with equation 2: $P = A + 6M$. Let's plug that into equation 1. So equation 1 becomes: $3A + (A + 6M) = 10M$. Which simplifies to $4A + 6M = 10M$ Now subtract 6M from both sides: $4A = 4M$ Divide both sides by 4: $A = M$ Wait, so an apple weighs the same as a plum? That's interesting. So if $A = M$, then from equation 2, $P = A + 6M$. Since $A = M$: $P = M + 6M = 7M$ So a pear is equal to 7 plums? But let me check if this makes sense with the first equation. Original equation 1: $3A + P = 10M$. If $A = M$ and $P = 7M$, then substituting in: $3M + 7M = 10M$. That works out. And equation 2: $A + 6M = P$ $M + 6M = 7M$ $7M = P$. Which also checks out. Wait, so according to this, one pear is equivalent to 7 plums. That seems straightforward, but let me make sure I didn't skip any steps or make any miscalculations. Starting again, the equations: 1) $3A + P = 10M$ 2) $P = A + 6M$ Substitute equation 2 into equation 1: $3A + (A + 6M) = 10M$ Which is $4A + 6M = 10M$ $6M = 4A = 4M$, so $A = M$ Then $P = A + 6M = 7M$. Yes, that's correct. So the equations are satisfied. Both are satisfied with $A = M$ and $P = 7M$. Therefore, one pear is equal to 7 plums. So the answer should be 7 plums balance one pear. But let me just visualize it again to be sure. Suppose a plum is a plum, then: Original equation: $1A + P = M + 6M = 7M$. Then $3A + P = 3M + 7M = 10M$, which is 10 plums. Correct. And $1A + 6M = M + 6M = 7M$, which is one pear. Perfect. So all checks out. Therefore, the answer is 7 </think> 7</p>
<p>Question:</p> <p>Given $n = 7^3 \times 11^2 \times 13^4$, find the number of integers that are divisors of n.</p>	<p><think> Okay, so I need to find the number of divisors of the number n which is given as 7 cubed times 11 cubed times 13 to the fourth power. Hmm, let's see I remember that there's a formula for finding the number of divisors when you have the prime factorization of a number. Let me try to recall how that works. First off, if a number is expressed as a product of primes raised to exponents various exponents, like $n = p_1^a \times p_2^b \times p_3^c \dots$ then the total number of divisors is $(a+1)(b+1)(c+1)\dots$ and so on for each prime factor. Is that right? I think I think so. Because for each prime factor, the exponent can range from 0 up to the exponent in the original number. So, for each prime, you have (ponent + 1) choices. Then you multiply them all together because they're independent choices. Let me verify that with a simple example. Suppose n is $2^2 \times 3^1$. Then the divisors are 1, 2, 3, 4, 6, 12. That's 6 divisors. Using the formula, $(2+1) \times (1+1) = 3 \times 2 = 6$. Yeah, that checks out. Another example: if n is $2^1 \times 5^1$. Then the divisors are 1, 2, 5. The formula is $(2+1) \times (1+1) = 3 \times 2 = 6$. Wait, that's not right. Let me recalculate. For $2^1 \times 5^1$, the divisors are 1, 2, 5, 10. That's 4 divisors. The formula is $(1+1) \times (1+1) = 2 \times 2 = 4$. Okay, that's correct. So I think the formula is solid. Alright, so applying this to the given n. The prime factors are 7, 11, and 13, each. to exponents are 3, 2, and 4 respectively. Therefore, the number of divisors should be $(3 + 1)(2 + 1)(4 + 1)$. Let's compute that. First, calculate each exponent one by one. For the 7: $3 + 1 = 4$. Then the 11 is 2, so $2 + 1 = 3$. Then 13 is 4, so $4 + 1 = 5$. Now multiplying those together: $4 \times 3 \times 5$ is 12. 12 times 5 is 60. So the number of divisors is 60? That seems straightforward. Wait, but, just to make sure I didn't make a mistake in the exponents. Let me check the original problem again. The given n is $7^3 \times 11^2 \times 13^4$. So, exponents are 3, 2, 4. Therefore, adding 1 to each exponent gives 4, 3, 5. Multiplying those together gives $4 \times 3 \times 5 = 60$. Yep, that's correct. So I guess there's not much more to it. It's a straightforward application of the number of divisors formula when the number is already in its prime factorized form. Since the problem is already in that form, it's just matter of applying the formula directly without having to factorize the number first. So that's easier. If the number wasn't already factorized, we would have to factorize it first, but here, it's already done. Let me just think if there's any way this could go wrong. Maybe if the exponents were read? For example, if someone thought 7^3 is just 7×3 instead of 7 cubed, but the problem uses the caret notation, so 7^3 is 7 cubed. Same with the others. So 11^2 is 11 squared, and 13^4 is 13 to the fourth. So the exponents are 3, 2, 4. So, that part seems okay. Alternatively, could there be a miscalculation? Let's compute $4 \times 3 \times 5$. Yes, 4×3 is 12, and 12×5 is 60. Seems solid. </think> 60</p>
<p>Question:</p> <p>If a natural number has an odd number of divisors, we call this number a "Hope Number". What is the largest "Hope Number" within 1000?</p>	<p><think> Okay, so I need to find the largest "Hope Number" within 1000. A Hope Number is a natural number that has an odd number of divisors. Let me start by recalling some math some concepts about Hope numbers. First, I remember that the number of divisors a number has is related to its prime factorization. If a number n can be factored into primes like $n = p_1^{a_1} p_2^{a_2} \dots p_k^{a_k}$, then the total number of divisors is $(a_1 + 1)(a_2 + 1) \dots (a_k + 1)$. Each exponent is increased by 1 because for each prime, the exponent in the divisor can range from 0 up to a_i. So for the number of divisors to be odd, each term in that product $(a_i + 1)$ must be odd. Because even one of them even is even, the product becomes even. So, each $(a_i + 1)$ must be odd. That implies that each a_i factor is even, which means the number must be a perfect square. Because if all exponents in the factorization are even, then the number is a square of some integer. Wait, let me verify that. Let's take the number 4. Its prime factorization is 2^2. The number of divisors is $(2 + 1) = 3$, which is odd. Another example, like 9: its prime factorization is 3^2, so it has $(2 + 1) = 3$ divisors. How about 6? Prime factors are $2^1 \times 3^1$. Number of divisors is $(1 + 1)(1 + 1) = 4$, which is even. So yes, only perfect squares have an odd number of divisors. Therefore, all Hope Numbers within 1000 are the perfect squares less than or equal to 1000. Therefore, the problem reduces to finding the largest perfect square less than or equal to 1000. Let me compute that. The square root of 1000 is approximately 31.627776. So the largest integer whose square is less than or equal to 1000 would be 31, since $31^2 = 961$, and $32^2 = 1024$, which is over 1000. Wait, hold wait, 31^2 is 961. Let me check that. 30^2 is 900, 31^2 is 961, 32^2 is 1024. So 1024 is over 1000, so 31^2 is 961. Therefore, 961 is the largest perfect square less than 1000. Therefore, 961 should be the largest Hope Number within 1000. But wait a minute, let me confirm that 961 is indeed a Hope Number. Let's check the prime factorization of 961. The prime factorization of 961 is 31^2. So its divisors are 1, 31, and 961. That's three divisors, which is odd. So yes, that checks out. But just to be thorough, let's make sure there are no perfect squares between 961 and 1000. The square of 31 is 961, and 32^2 is 1024, which is over 1000. So integers are between 31 and 32, so there are no perfect squares between 961 and 1000. Therefore, 961 is indeed the largest one. Alternatively, maybe I made a mistake that only perfect squares have an odd number of divisors. Let me verify with another example. Let's take 16. Its divisors are 1, 2, 4, 8, 16. That's five divisors, which is odd. 25 is 5^2, 25 squared. How about 9? Divisors: 1, 3, 9, three divisors. 10: 1, 2, 5, 10. Four divisors. So yes, only squares of integers have odd divisor counts. Therefore, my conclusion holds. Therefore, the largest Hope Number within 1000 is 961. </think> 961</p>