

Question: If a natural number has an odd number of divisors, we call this number a "Hope Number". What is the largest "Hope Number" within 1000?

Show-o & Emu3 & Janus Pro 7B



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do not support this task.**

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Okay, so I need to find the largest "Hope Number" within 1000. A Hope Number is a natural number that has an odd number of divisors. Let me start by recalling some math concepts about Hope numbers.

First, I remember that the number of divisors a number has is related to its prime factorization. If a number n can be factored into primes like $n = p_1^{a_1} p_2^{a_2} \dots p_k^{a_k}$, then the total number of divisors is $(a_1 + 1)(a_2 + 1) \dots (a_k + 1)$. Each exponent is increased by 1 because for each prime, the exponent in the divisor can range from 0 up to a_i . So for the number of divisors to be odd, each term in that product $(a_i + 1)$ must be odd. Because even one of them even is even, the product becomes even. So, each $(a_i + 1)$ must be odd. That implies that each a_i factor is even, which means the number must be a perfect square. Because if all exponents in the factorization are even, then the number is a square of some integer.

Wait, let me verify that. Let's take the number 4. Its prime factorization is 2^2 . The number of divisors is $(2 + 1) = 3$, which is odd. Another example, like 9: its prime factorization is 3^2 , so it has $(2 + 1) = 3$ divisors. How about 6? Prime factors are $2^1 * 3^1$. Number of divisors is $(1 + 1)(1 + 1) = 4$, which is even. So yes, only perfect squares have an odd number of divisors. Therefore, all Hope Numbers within 1000 are the perfect squares less than or equal to 1000.

Therefore, the problem reduces to finding the largest perfect square less than or equal to 1000. Let me compute that.

The square root of 1000 is approximately 31.627776. So the largest integer whose square is less than or equal to 1000 would be 31, since $31^2 = 961$, and $32^2 = 1024$, which is over 1000. Wait, hold wait, 31^2 is 31^2 . Let me check that. 30^2 is 900, 31^2 is 961, 32^2 is 1024. So 1024 is over 1000, so 31^2 is 961. Therefore, 961 is the largest perfect square less than 1000. Therefore, 961 should be the largest Hope Number within 1000.

But wait a minute, let me confirm that 961 is indeed a Hope Number. Let's check the prime factorization of 961. The prime factorization of 961 is 31^2 . So its divisors are 1, 31, and 961. That's three divisors, which is odd. So yes, that checks out.

But just to be thorough, let's make sure there are no perfect squares between 961 and 1000. The square of 31 is 961, and 32^2 is 1024, which is over 1000. So integers are between 31 and 32, so there are no perfect squares between 961 and 1000. Therefore, 961 is indeed the largest one.

Alternatively, maybe I made a mistake that only perfect squares have an odd number of divisors. Let me verify with another example. Let's take 16. Its divisors are 1, 2, 4, 8, 16. That's five divisors, which is odd. 25 is 5^2 , 25 squared. How about 9? Divisors: 1, 3, 9, three divisors. 10: 1, 2, 5, 10. Four divisors. So yes, only squares of integers have odd divisor counts.

Therefore, my conclusion holds.

Therefore, the largest Hope Number within 1000 is **961**.

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961